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By

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

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The Statics Ratio for Analysis of Frames that Deflect

Synopsis

A general method for analyzing frames subject to several degrees of freedom of movement or deflection is proposed for practical use. The method operates upon an identical frame as a model. The model frame is arbitrarily deflected by trial increments until it is reasonably similar in its deflection pattern to that of the loaded frame. The arbitrary deflections are represented by simple moment distribution methods. Statics ratios, across particular sections of the frame, relate the resisting shears (or moments) on the deflected model to the external shears on the loaded frame. These ratios furnish the chief tool for the analysis. They determine the need for corrective deflections of the model and also the magnitude of these corrections; and finally they determine the degree of proportionality that results. True moments are determined by dividing the final model moments by the statics ratios. The method is shown to apply to trapezoidal panels as well as to rectangular. Special deflection patterns are developed to facilitate the solution of trapezoidal panels.

Symbols and Signs

V_1 = external shear acting on section 1–1 of loaded frame.

V_{AB} = shear carried by member AB of model frame.

M_{AB} = moment acting on joint A from member AB of model frame (equal numerically to bending moment at A end of member AB of the model frame).

SR_1 = statics ratio on section 1–1, i.e., ratio of resisting shear of the members on section 1–1 across the model frame to the external shear on the same section through the loaded frame; also formulated in terms of resultant moments or forces instead of shears.

$M_{f,\,AB}$ = fixed-end moment acting on joint A from member AB. (Some may be accustomed to using $M^F{}_{AB}$ for this quantity.)

$\triangle, \triangle'$ = deflection of a point.

ϕ = angle between a tower leg and the vertical.

θ = angle through which a joint rotates; positive when counterclockwise.

Sign Convention

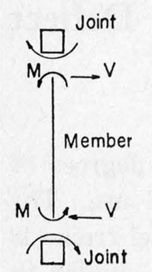


FIGURE 1

Positive moments will be defined as those which tend to rotate the *joint* clockwise. Shears carry the standard signs. All the moments and shears shown in Fig. 1 are positive.

Introduction

When moment distribution as a method of frame analysis began to displace more formal solutions involving simultaneous equations, it retained a certain degree of formality in its approach. A fixed sequence of steps was natural: (1) fixed-end moments for each loaded member; (2) distributed moments resulting from unbalance at joints; (3) carry-over moments to adjacent joints; (4) further cycles of distribution and carry-over as needed. Under such procedure the beginner and expert go through the same motions and get the same answers in the same number of steps. Their work differs only in the efficiency and accuracy with which the expert introduces and defines artificial boundaries in order to reduce the number of joints involved; and to some extent in the fact that the expert does not feel bound to a definite sequence of joint releases.

When side-sway or frame deflection is important, the formal approach has not led to such efficient procedures except in the case of one story frames. Frames free to deflect laterally at more than one story level or to deflect vertically at more than one panel point, or with equivalent freedoms in any direction or directions, may be said to have more than one degree of freedom of movement. For these frames, the method of successive corrections and the method of simultaneous equations (based upon influence deflections) are both used. The first of these methods is inherently lengthy. The second method becomes lengthy as the number of degrees of freedom of movement is increased; and the solving of simultaneous equations seems contrary to the general philosophy of moment distribution.

In 1933 Dr. L. E. Grinter proposed[1] a "simplified method" for wind stresses which in a limited field used moment distribution in a less restricted manner. He *estimated* deflections, measured the accuracy of his results with a shear ratio, and corrected them if necessary by writing additional fixed-end moments. He stated that the method also had value in analyzing single story bents and Vierendeel trusses. He did not explain in detail the philosophy behind this approach, especially the problem of correction moments, with the result that the method has not received

[1]"Wind Stress Analysis Simplified," by L. E. Grinter, Member ASCE, Transactions ASCE, Vol. 99 (1934), p. 610.

as much attention as it deserves. Some erroneously considered this method semi-empirical, for a special usage where only a fair degree of accuracy was important.

A general method of procedure is here proposed which has a wide field of application for structures involving sidesway or deflection. Dr. Grinter's "simplified method" could be regarded as a special case within this general field. This general method eliminates the need for simultaneous equations. In many cases, especially complex ones, it is the simplest method yet proposed. Any degree of mathematical accuracy desired can be obtained, but an ordinary slide rule is adequate for most practical usage, and a six-inch slide rule was used for most of the sample calculations that follow. As in other moment distribution calculations, the particular problem can be stopped when any desired degree of accuracy has been achieved. Since the method has some of the elements of a trial-and-error process, an operator becomes more skillful with practice; and a skillful operator can shorten the process considerably. Nevertheless, the procedure does not require skill; it automatically points the way for each additional trial.

The Model Frame Concept

The idea of a model frame, which will be defined here simply as an identical frame dissociated from the given loading and then arbitrarily deflected or displaced, is a useful one in complex situations. In Fig. 2a, a given system of forces causes a unique set of moments, shears, and deflections. If the corresponding model frame is arbitrarily displaced as in Fig. 2b, the chances are that the deflections $\triangle_1$, $\triangle_2$, and $\triangle_3$ do not bear any constant ratio to the corresponding deflections of Fig. 2a. How-

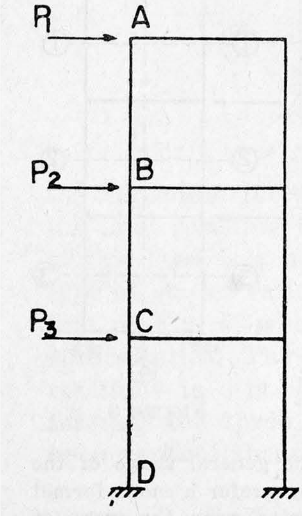


FIGURE 2a

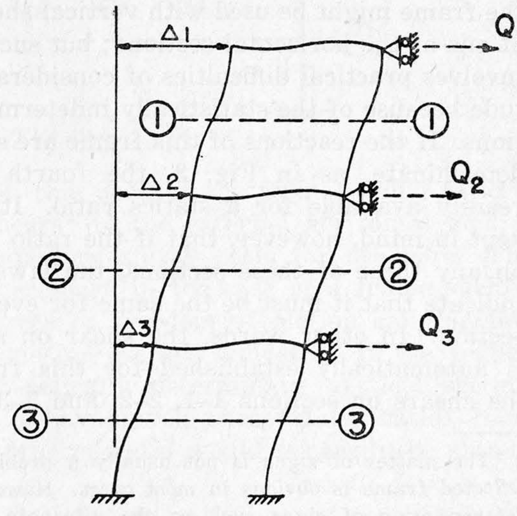


FIGURE 2b

ever, if identical ratios happen to exist,* Fig. 2b is a true or correct model of the frame loaded as in Fig. 2a; that is, the stresses of Fig. 2a can be obtained from those of the model simply by dividing by this common ratio. Since proportional moments, shears, loads, deflections, reactions must all go together (within the proportional limit of the materials), any of these may be taken as measures of proportionality or correct model action. Until there is proportionality between a loaded frame and its model, there is no very useful theoretical relationship between them. This paper is concerned with methods of producing and measuring proportionality in cases where there is more than one degree of freedom of motion. However, it is also shown that approximate proportionality is often adequate for ordinary use.

Statics Ratios as a Measure of Proportionality

In a frame of the type of Fig. 2, a convenient measure of proportionality lies in the *statics ratio,* that is, the ratio of the resisting shear of the members at any level in the model to the external shear at the same level in the loaded frame. These resisting shears at the several levels in the model would be sufficient to define the holding forces Q_1, Q_2, and Q_3. The number of such shears required is a simple matter of statics. Three shears (on independent sections) can define three holding forces; n shears are required for a frame having n degrees of freedom of motion and hence n holding forces. Thus the minimum number of statics ratios to be set up must equal the number of degrees of freedom of motion of the frame.

In Fig. 2b the simplest sections to use are the horizontal sections shown across the vertical panels. Theoretically a vertical section down the middle of the frame might be used with vertical shears in lieu of one of the horizontal sections; but such a section involves practical difficulties of considerable magnitude because of the statistically indeterminate reactions. If the reactions of this frame are statistically determinate, as in Fig. 3, the fourth section is readily available for a statics ratio. It should be kept in mind, however, that if the ratio is identical on any three of these sections, the laws of statics indicate that it must be the same for every possible section. In other words, the shear on section 4–4 is automatically established for this frame when the shears on sections 1–1, 2–2, and 3–3 are fixed.

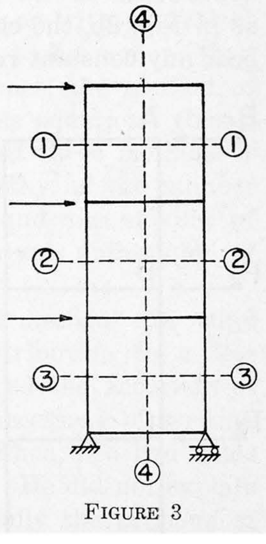


FIGURE 3

*The matter of signs is not usually a problem because the general shape of the deflected frame is obvious in most cases. However, some may prefer a more formal determination of signs such as the alternate process indicated near the start of Example III.

The same is true of Fig. 4a. Nevertheless, some practical advantages often accrue from the use of an extra section, as in Example I.

External shears have been discussed for the statics ratio purely as a matter of convenience. In the frame of Fig. 4a, the difference between shears on sections 1–1 and 2–2 is equivalent to the sum of the horizontal forces on the free body AB of Fig. 4b (here simplified by showing only

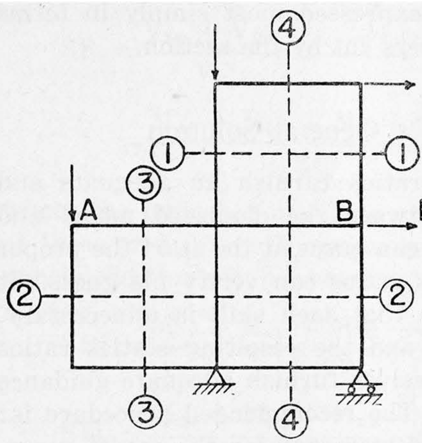


FIGURE 4a

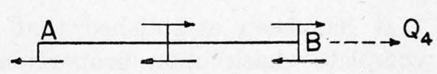


FIGURE 4b

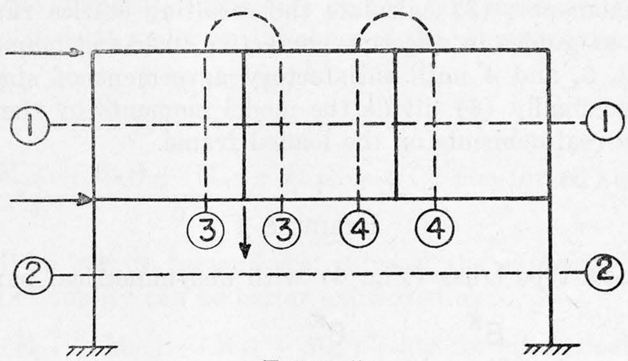


FIGURE 5

the horizontal forces that act on this deck of the frame). The ratio of the total resisting horizontal shears* to the external horizontal load P_4 at this level is a perfectly satisfactory statics ratio for use here. This type of statics ratio is almost necessary in the case of a frame such as that of Fig. 5, where four degrees of freedom of motion call for four statics ratios. The logical sections for use are indicated. If the vertical reactions in Fig. 5 had been statically determinate, vertical sections through the three panels would have given three static ratios based upon vertical shears; and this type of ratio would generally be simpler

*The resisting horizontal shears are equal in magnitude to force Q_4 but opposite in direction.

to use. Of course, this would have provided one more ratio than the minimum required.

In general, there must be enough equations of statics involved to make sure that the external forces on the model are actually proportional to those in the given frame. But one has as much liberty in setting up the form of these statics equations as he has in any problem of statics. It will be shown later that moment equations are sometimes the simplest form. Also, resisting shears are often expressed most simply in terms of the end moments on the several members cut by the section.

Statics Ratios as Guides to a General Solution

It has been established that statics ratios furnish an adequate and complete check upon proportionality between the displaced model and the loaded frame. If the skilled designer can guess at the start the proper relative displacements to use, the statics ratios can verify his guess. It will now be shown by several examples that such skill is unnecessary. Even when initial estimates are crude and the resulting statics ratios vary widely, these divergent ratios themselves furnish adequate guidance for correcting the model displacements. The recommended procedure is: (1) estimate or guess at initial fixed-end moments for the model corresponding to some reasonable deflection pattern; (2) distribute and balance these moments; (3) calculate the resulting statics ratios; (4) use these ratios as guides in selecting corrective fixed-end moments; (5) repeat steps 2, 3, and 4 until satisfactory agreement of statics ratios is obtained; and finally (6) divide the model moments by the statics ratio to obtain the real moments on the loaded frame.

Example I

A Vierendeel type truss (Fig. 6) with unsymmetrical panel loads and

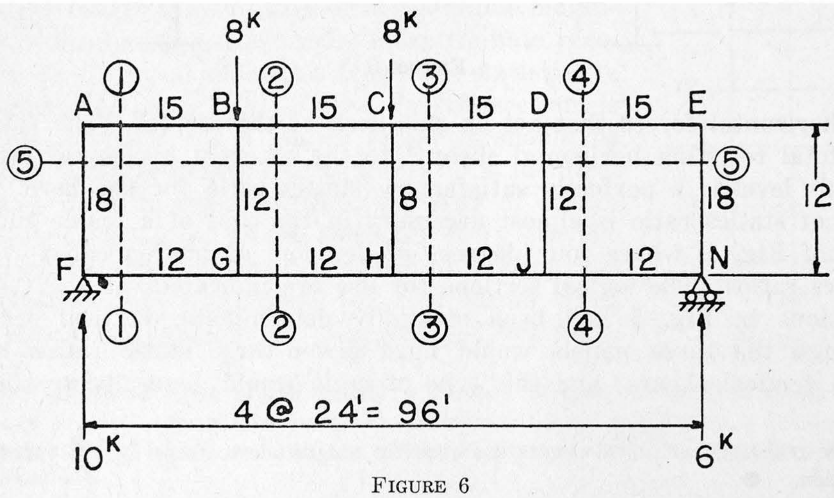


FIGURE 6

stiffnesses as indicated on each member will be analyzed for all joint moments. Axial shortening of members will be neglected here.

Shears on sections 1–1, 2–2, 3–3, and 4–4 will be used to establish statics ratios. Section 5–5 will also be used, but since there are only four degrees of freedom, the use of this section is not actually required. On section 1–1 the statics ratio is

$$SR_1 = \frac{V_{AB} + V_{FG}}{V_1} = \frac{\dfrac{M_{AB} + M_{BA}}{24} + \dfrac{M_{FG} + M_{GF}}{24}}{+10}$$

$$= \frac{M_{AB} + M_{BA} + M_{FG} + M_{GF}}{+240}$$

where the indicated moments are those in the deflected model. The total of these moments will obviously be positive under this loading since the panel deflection will be somewhat related to that shown in Fig. 7. Similarly

$$SR_2 = \frac{M_{BC} + M_{CB} + M_{GH} + M_{HG}}{+48}$$

$$SR_3 = \frac{M_{CD} + M_{DC} + M_{HJ} + M_{JH}}{-144}$$

The total of the numerator moments of SR_3 will necessarily be negative when the panel deflects under the influence of the negative external shear.

$$SR_4 = \frac{M_{DE} + M_{ED} + M_{JN} + M_{NJ}}{-144}$$

$$SR_5 = \frac{(M_{AF} + M_{FA}) + (M_{BG} + M_{GB}) + \ldots \text{ etc. for all verticals}}{0}$$

Since SR_5 will be infinite for any real value of the numerator, this condition for proportionality can be better expressed as:

$$\Sigma M_{5-5} = (M_{AF} + M_{FA}) + (M_{BG} + M_{GB}) + \text{etc. for all verticals} = 0$$

This statement of the condition is not directly comparable with the other SR values but this does not interfere with its general usefulness.

Two types of deflection patterns will be used in this example to build up the necessary deflections. Pattern #1, which will be used on all sections except 5–5, represents a relative vertical deflection in any single panel, with all joints fixed against rotation, as in Fig. 7. The relative fixed-end moments are noted alongside each deflected member, those of the top chord being greater in the ratio of the relative stiffness values, i.e., $\frac{15}{12} \times 1.00 = 1.25$. This pattern

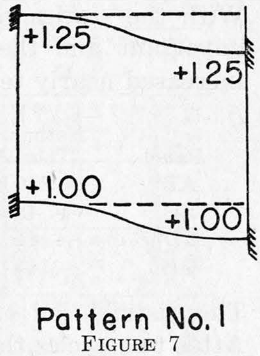
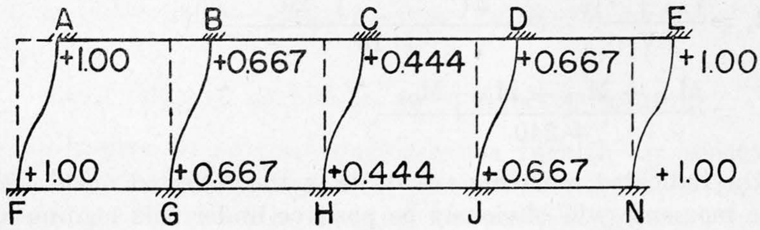


FIGURE 7

#1 can be applied to any panel since the relative chord stiffness is repeated in each panel. Pattern #2 (Fig. 8) corresponds to a lateral deflection of the entire top chord, without joint rotation. The relative moments are proportional to relative stiffness of verticals and are recorded conveniently on this figure. The absolute amount of the deflection in either pattern is not important because it is more convenient to measure deflection in terms of fixed-end moments as shown.



Pattern No. 2

FIGURE 8

To secure initial trial deflections and moments that are somewhat related to the given loads, initial moments will be estimated as directly proportional to the panel shear and the panel length. For instance, in panel AB the shear is $+10^k$, the panel length is 24′, and $M_{AB} + M_{BA} + M_{FG} + M_{GF}$ should total $10 \times 24 = +240$ units. On this basis the following total moments are found:

Panel	V × Panel Length
AB	$+10 \times 24 = +240$
BC	$+2 \times 24 = +48$
CD	$-6 \times 24 = -144$
DE	$-6 \times 24 = -144$

There is no point in using exactly these values. Experienced designers will see several refinements that could very properly be introduced here for a better guess as to starting moments, refinements that would correspond to different relative deflections.

For slide rule work, it is convenient to have minimum initial moments as large as 500 to 1000 since this makes it unnecessary to record decimals. With the statics ratio method, relative rather than absolute values are important and the ones indicated for use below have been arbitrarily increased nearly ten times.

Panel	Estimated Total M	Use Pattern #1		Bottom M_f	Top M_f	Total
AB	+ 240 kf	+ 500 units	=	+ 500	+ 625	+ 2250
BC	+ 48	+ 100	=	+ 100	+ 125	+ 450
CD	— 144	— 300	=	— 300	— 375	— 1350
DE	— 144	— 300	=	— 300	— 375	— 1350

The distribution of these initial fixed-end moments is shown in Fig. 9. After two cycles the moments are summed (excluding the last unbalanced

	A		B			C			D			E	
	AF	AB	BA	BG	BC	CB	CH	CD	DC	DJ	DE	ED	EN
	0.545	0.455	0.357	0.286	0.357	0.395	0.210	0.395	0.357	0.286	0.357	0.455	0.545
F	0	+625	+625		+125	+125		-375	-375		-375	-375	
D	-341	-284	-267	-215	-268	+99	+52	+99	+267	+215	+268	+171	+204
C	-150	-134	-142	-100	+50	-134	+25	+134	+50	+100	+86	+134	+90
D	+155	+129	+68	+55	+69	-10	-5	-10	-84	-68	-84	-102	-122
Σ1	-336	+336	+284	-260	-24	+80	+72	-152	-142	+247	-105	-172	+172
C	+81	+34	+64	+28	-5	+34	-3	-42	-5	-34	-51	-42	-60
F	+20	-188	-188	+13			+9	+38	+38	+13	+50	+50	+20
Σ2	-212	+212	+198	-188	-10	+112	+64	-176	-120	+223	-103	-155	+155
C	-7	-3	-5	-2	-2	-2	0	0	-2	0	-2	0	-2
F	-8	+27	+28	-5			-4	+20	+20	-5			-8
Σ3	-232	+232	+216	-199	-17	+104	+57	-161	-106	+214	-108	-150	+150
C	-2	-2	-2	-2	-3	-2	-1	-2	-2	-2	+2	-2	+3
F	+3	-2	-3	+2	+2	+3	+1	+2	+3	+2			+3
Σ4	-229	+229	+213	-197	-16	+105	+57	-162	-105	+213	-108	-154	+154
FINAL	-64.9	+64.9	+60.4	-55.9	-4.5	+29.8	+16.1	-45.9	-29.7	+60.1	-30.4	-43.4	+43.4

	F		G			H			J			N	
	FA	FG	GF	GB	GH	HG	HC	HJ	JH	JD	JN	NJ	NE
	0.600	0.400	0.333	0.334	0.333	0.375	0.250	0.375	0.333	0.334	0.333	0.400	0.600
F	0	+500	+500		+100	+100		-300	-300		-300	-300	
D	-300	-200	-200	-200	-200	+75	+50	+75	+200	+200	+200	+120	+180
C	-170	-100	-100	-108	+38	-100	+26	+100	+38	+108	+60	+100	+102
D	+162	+108	+56	+57	+57	-10	-6	-10	-68	-69	-69	-81	-121
Σ1	-308	+308	+256	-251	-5	+65	+70	-135	-130	+239	-109	-161	+161
C	+78	+28	+54	+28	-5	+28	-2	-34	-5	-34	-40	-34	-61
F	+20	-150	-150	+13			+9	+30	+30	+13	+40	+40	+20
Σ2	-196	+196	+182	-182	0	+89	+64	-153	-113	+215	-102	-144	+144
C	-6	-2	-5	-2	-1	-2	-1	0	-1	0	-2	0	-2
F	-8	+22	+22	-5			-4	+16	+16	-5			-8
Σ3	-214	+214	+196	-192	-4	+84	+57	-141	-100	+207	-107	-140	+140
C	-2	-2	-1	-2	-2	-2	-2	-1	-2	-2	+2	-1	+2
F	+3	-2	-2	+2	+2	+2	+1	+2	+2	+2			+3
Σ4	-211	+211	+194	-190	-4	+84	+56	-140	-101	+207	-106	-143	+143
FINAL	-59.8	+59.8	+55.0	-53.9	-1.1	+23.8	+15.9	-39.7	-28.7	+58.6	-29.9	-40.3	+40.3

FIGURE 9

carry-over moments) and the first trial statics ratios are calculated as follows:

$$SR_1 = \frac{+336 + 284 + 308 + 256}{+240} = \frac{+1184}{+240} = +4.93$$

$$SR_2 = \frac{-24 + 80 - 5 + 65}{+48} = \frac{+116}{+48} = +2.42$$

$$SR_3 = \frac{-152 - 142 - 135 - 130}{-144} = \frac{-559}{-144} = +3.88$$

$$SR_4 = \frac{-105 - 172 - 109 - 161}{-144} = \frac{-547}{-144} = +3.80$$

$$\Sigma M_{5-5} = -336 - 308 - 260 - 251 + 72 + 70 + 247 + 239 + 172 + 161 = -194$$

It is necessary to bring these statics ratios into better agreement, but one considerable advantage of this procedure is that it does not matter upon what value they converge. A statics ratio of 3.80 looks like as simple a value as any to attempt next. This obviously indicates the addition of negative moments (pattern #1) to section 1–1, the addition of positive moments (pattern #1) to section 2–2, only slight, if any, changes on sections 3–3 and 4–4, and the addition of positive moments (pattern #2) to section 5–5. The operator can guess at the amounts to use, or he can use preliminary calculations, either rough or refined, involving: the amount of the ratio change desired; the existing (last) carry-over moments not yet included in the summation; the distribution factors involved; and the indirect effect of other additions in adjacent panels. Exact consideration of all these items would involve troublesome simultaneous equations, but approximate estimates are a powerful tool in the hands of a designer experienced in moment distribution. Simpler methods will be illustrated here, involving the desired change in the statics ratio, the existing unbalanced carry-over moment, and only a guess at something extra to allow for "shrinkage" due to distribution. Ordinarily approximate mental arithmetic would be used here, but the following table is added to show the method in more detail.

SR No.	Needed Ratio Change	Change in SR Numerator	Change in ΣM for Sect.	Unbal. Carry-Over M	Total M to Add	Allowing for Shrinkage: Add	Pattern	Total
1	— 1.13	— 271	— 271	+ 180	— 451	— 150 units	#1	— 676
2	+ 1.38	+ 66	+ 66	+ 52	+ 14	0		0
3	— 0.08	+ 12	+ 12	— 86	+ 98	+ 30	#1	+ 136
4	0	0	0	— 167	+ 167	+ 40	#1	+ 180
5	....	....	+ 194	+ 21	+ 173	+ 20	#2	+ 151

No tabulation is needed once the general idea is clearly grasped.

These added moments, after two cycles of distribution, lead to new statics ratios. The detail of these distributions has been omitted from Fig. 9 because of limited room, but the resulting totals and unbalanced carry-over are shown. The second trial statics ratios become:

$$SR_1 = \frac{+788}{+240} = +3.27$$

$$SR_2 = \frac{+191}{+48} = +3.97$$

$$SR_3 = \frac{-562}{-144} = +3.90$$

$$SR_4 = \frac{-504}{-144} = +3.50$$

$$\Sigma M_{5-5} = +87$$

These are in better agreement than the first trials but SR_1 and SR_4 are not entirely satisfactory. When new statics ratios fail thus to respond nearly as expected, something can often be learned from looking over the distribution calculations. Sometimes errors in signs or arithmetic are thus noted; sometimes, as in this case, the error in one's earlier judgment is revealed. In the case of SR_1 the use of an extra 50% allowance for "shrinkage" was decidedy too much and caused an over-correction of this ratio. The SR_4 correction moments incuded only a small "shrinkage" allowance and the reduction in SR_4 just found came from some moments on member EN. New moment additions will now be found to adjust the statics ratios to about 3.50 (3.90 would be equally as logical).

SR No.	Needed Ratio Change	Change in SR Numerator	Change in ΣM for Sect.	Unbal. Carry-Over M	Total M to Add	Allowing for Shrinkage Add	Pattern	Total
1	+ 0.23	+ 55	+ 55	— 15	+ 70	+ 22 units	#1	+ 99
2	— 0.47	— 22	— 22	— 7	— 15	0		
3	— 0.40	+ 58	+ 58	— 3	+ 61	+ 16	#1	+ 72
4	0	0	— 0	— 4	+ 4	0		
5	----	----	— 87	— 22	— 65	— 8	#2	— 60

One cycle of distribution then leads to the third trial statics ratios and further corrections:

$$SR_1 = \frac{+232+216+214+196}{+240} = \frac{+858}{+240} = 3.57$$

$$SR_2 = \frac{-17+104-4+84}{+48} = \frac{+167}{+48} = 3.48$$

$$SR_3 = \frac{-161-106-141-100}{-144} = \frac{-508}{-144} = +3.52$$

$$SR_4 = \frac{-108-150-107-140}{-144} = \frac{-505}{-144} = +3.50$$

$$\Sigma M_{5-5} = -232-214-199-192+57+57+214+207+150+140 = -12$$

SR No.	Needed Ratio Change	Change in SR Numerator	Change in ΣM for Sect.	Unbal. Carry-Over M	Total M to Add	Allowing for Shrinkage Add	Pattern	Total
1	−0.07	− 17	− 17	− 7	− 10	− 2 units	#1	− 9
2	+ 0.02	+ 1	+ 1	− 9	+ 10	+ 2	#1	+ 9
3	− 0.02	+ 3	+ 3	− 7	+ 10	+ 2	#1	+ 9
4	0	0	0	+ 1	− 1	0	----	0
5	----	----	+ 12	− 10	+ 22	+ 3	#2	+ 22

Two cycles of distribution bring the model into equilibrium. The fourth (final) statics ratios become:

$$SR_1 = \frac{+229+213+211+194}{+240} = \frac{+847}{+240} = 3.53$$

$$SR_2 = \frac{-16+105-4+84}{+48} = \frac{+169}{+48} = 3.52$$

$$SR_3 = \frac{-162-105-140-101}{-144} = \frac{-508}{-144} = 3.53$$

$$SR_4 = \frac{-108-154-106-143}{-144} = \frac{-511}{-144} = 3.55$$

$$\Sigma M_{5-5} = -229-211-197-190+57+56+213+207+154+143 = +3$$

If all statics ratios were identical the final solution would be given by dividing all moments by this one ratio. Although this condition essentially exists in these ratios, an appropriate procedure is used here that seems to give better results, especially when the statics ratios show more spread. Moments on any member cut by one of the statics ratio sections are found by dividing the final model moments by the particular statics ratio for that section. Moments for members not cut by any section are taken of such size as to balance the joint moments. These final moments are shown as the last line of data in Fig. 9. (It might be noted here that Fig. 9 shows the complete tabulation except for seven omitted lines of distribution and carry-over moments.)

Accuracy of Results

There is no real purpose served in most cases by securing such close agreement as above between statics ratios. In Example I, it was desired to show that exact convergence upon some common value was a relatively simple matter. As to accuracy, it can always be said that the last digit recorded in any moment distribution process may be in error by one or even two units. If one wants mathematical accuracy in the unit column, he must tabulate at least one decimal place. Slightly more than the usual error from discarded fractions is possible here because of the separate addition of several small increments; but this difference is not of much practical significance. A separate check, starting again with the total fixed-end moments and then distributing until balanced, is a very good practice in order to locate and eliminate any real errors in arithmetic or signs; and such a check eliminates the errors due to many small increments. Accuracy in this example has also been limited by the use of a six-inch slide rule.

Reasonable accuracy seems to result from statics ratios only reasonably in agreement. In practical work, it would seem to be unnecessary in many cases to secure an agreement between adjacent statics ratios closer than 10% ; and in many cases a further spread might be adequate. There seems to be no real purpose served in a practical problem by securing agreement between adjacent statics ratios closer than 3 or 4%. Dr. Grinter has already pointed out[2] in his closely related study of wind stresses in tall building frames that his criterion ratios (for the above example the same as the inverse of the statics ratios) may differ by 10% in adjacent panels with final moment errors seldom more than 5 or 6%.

Table I for Example I, and Tables II and III for later examples, have been prepared for further study of the accuracy obtained when the solution is stopped at various stages of agreement between statics ratios. The results of check calculations starting with the summation of fixed-end moments have been entered in the first line under the designation of "Recap. Values" and have been used as a reference base. (These values are themselves subject to errors of 0.3kf, and an occasional error of 0.6kf, corresponding to cumulative errors of 1 and 2 units, respectively, in the moment distribution process.) Below these values are tabulated final moments that would have been found by using various preliminary sets of statics ratios; and with each of these the per cent variation from the first line of data. For the first trial set of statics ratios (tabulated last in the table), the maximum statics ratio is more than twice the minimum and this occurs in adjacent panels. Yet the maximum error in moment is 22% except for four values that are numericaly small and not very important. For the second set of statics ratios, with a maximum spread of 19% (based on the average ratio) and an adjacent spread of

[2]Grinter, "Theory of Modern Steel Structures," Vol. II, p. 169.

TABLE I

RELATIVE ACCURACY USING DIFFERENT STATICS RATIOS

A B C D E

	AF	AB	SR	BA	BG	BC	SR	CB	CH	CD	SR	DC	DJ	DE	SR	ED	EN
RECAP. VALUES	-64.6	-64.6	3.53	+60.4	-55.9	- 4.5	3.54	+29.6	+16.1	-45.7	3.54	-29.9	+60.7	-30.8	3.53	-43.3	+43.3
FINAL #4	- 64.9	+64.9	3.53	+60.4	- 55.9	- 4.5	3.52	+29.8	+16.1	-45.9	3.53	-29.7	+60.1	-30.4	3.55	-43.4	+43.4
	+0.5%	+0.5%		0%	0%	0%		+0.7%	0%	+0.4%		-0.7%	-1.0%	-1.3%		+0.2%	+0.2%
#3	-64.9	+64.9	3.57	+60.4	-55.5	- 4.9	3.48	+29.9	+ 15.7	-45.6	3.52	-30.0	+60.8	- 30.8	3.50	-42.8	+42.8
	+0.5%	+0.5%		0%	-0.7%	+ 9%		+1.0%	-2.5%	-0.2%		+0.3%	+0.2%	0%		- 1.2%	- 1.2%
#2	- 64.6	+ 64.6	3.27	+ 60.3	- 57.8	- 2.5	3.97	+ 28.1	+ 17.0	-45.1	3.90	-30.7	+ 59.8	- 29.1	3.50	-44.3	+44.3
	0%	0%		-0.2%	+ 3%	-44%		- 5%	+ 6%	-1.3%		+2.7%	-1.5%	- 6%		+2.3%	+2.3%
#1	-68.2	+68.2	4.93	+57.5	-47.6	- 9.9	2.42	+33.0	+ 6.1	- 39.1	3.88	- 36.5	+64.2	- 27.7	3.80	- 45.3	+45.3
	+ 6%	+ 6%		- 5%	-15%	+120%		+ 11%	-62%	- 14%		+22%	+ 6%	- 10%		+ 5%	+ 5%

F G H J N

	FA	FG	GF	GB	GH	HG	HC	HJ	JH	JD	JN	NJ	NE
R. VALUES	- 60.1	+ 60.1	+ 55.0	- 53.9	- 1.1	+24.0	+ 15.5	-39.5	- 28.8	+58.5	-29.7	-40.2	+40.2
FINAL #4	-59.8	+59.8	+ 55.0	- 53.9	- 1.1	+23.8	+ 15.9	-39.7	-28.7	+ 58.6	- 29.9	-40.3	+40.3
	-0.5%	-0.5%	0%	0%	0%	-0.8%	+2.6%	+0.5%	-0.3%	+0.2%	+0.7%	+0.2%	+0.2%
#3	- 59.9	+ 59.9	+54.8	-53.7	- 1.1	+ 24.1	+ 15.9	-40.0	- 28.6	+ 59.1	-30.5	- 39.9	+39.9
	-0.3%	-0.3%	-0.4%	-0.4%	0%	+0.4%	+2.6%	+1.3%	-0.7%	+1.0%	+2.7%	-0.7%	-0.7%
#2	- 59.7	+ 59.7	+ 55.4	- 55.4	0	+ 22.4	+ 16.8	- 39.2	- 29.0	+ 58.4	- 29.4	- 41.2	+ 41.2
	-0.7%	-0.7%	-1.0%	+2.8%	-100%	- 7%	+ 8%	-0.8%	+0.7%	-0.2%	-1.0%	+2.5%	+2.5%
#1	- 62.5	+ 62.5	+ 51.8	- 49.7	- 2.1	+26.8	+ 7.9	-34.7	- 33.5	+62.2	- 28.7	-42.3	+ 42.3
	+ 4%	+ 4%	- 6%	- 8%	+ 91%	+12%	-49%	- 12%	+17%	+ 6%	- 3%	+ 5%	+ 5%

19%, the maximum moment error is 2.0kf and the maximum percentage error is 7% (except for one moment M_{BC} which is numerically very small). The third set of statics ratios, with a maximum spread of 2.6%, and an adjacent spread of 2.6%, gives a maximum error of 0.8kf which happens to be 2.7% error for that medium sized moment; one other small moment has a 9% error due to a 0.4kf difference, but only five values have errors greater than 0.4kf. Few calculations would warrant more exactness than given by the second set of statics ratios, almost none more than given by the third set. Stopping at either of these points would shorten the calculations considerably.

True Deflections

It should be noted that definite movements are involved in the writing of initial and added fixed-end moments. But these are relative movements, not final deflections relative to the original supports (except by chance). If one investigated the total movement of N relative to F, for the data used in solving Example I, he would find that N has been raised above its original position. For true deflections, the whole truss must then be rotated through a small clockwise angle about F to return N to its original level. This would give a correction to both vertical movements and sidesway. (The fixed-end moments written in on any vertical member measure the sidesway already introduced.)

If no fixed-end moments in the verticals (Section 5–5) had been written, a solution would have been entirely possible, although slightly slower in converging. Such a solution of this problem required six increments of fixed-end moments instead of the four used here for equal agreement between statics ratios. In such a procedure no sidesway is introduced and N is displaced even further relative to F. The entire sidesway in this case can then be visualized in terms of a rotation angle (of the entire frame about F) to return N to its original position.

Example II

This is an irregular frame (Fig. 10) and the three logical statics ratio sections shown are not entirely independent, each cutting one or more members also cut by other sections. Hence a deflection that increases the shear on one of these sections directly increases the shear on another section and makes it more difficult to foresee the entire effect of a movement. There are many different movements that can be used, in the sense that they are movements for which one can write the fixed-end moments without difficulty. Sometimes complex movements are helpful in this process of solution by trial, as will be demonstrated in Example III, but generally simple movements along the sections used in setting up the statics ratios will be the easiest to manipulate.

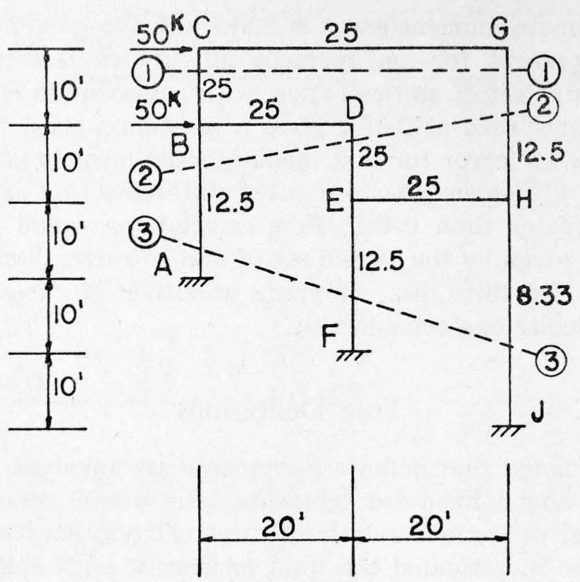


FIGURE 10

This frame has only three degrees of freedom of motion and many engineers may find the use of influence deflections and three simultaneous equations more to their liking. Statics ratios show to more advantage when the number of degrees of freedom is larger. This example is included to show the treatment of overlapping sections and members of unequal length.

On section 1–1, deflections as shown in Fig. 11a will be used, represented by fixed-end moments in the ratios recorded on the figure. Here the fixed-end moment varies as K/L or I/L². Since the larger moment is in member CB, a simple estimate of the starting fixed-end moment is based on a 50 kip shear on this member.

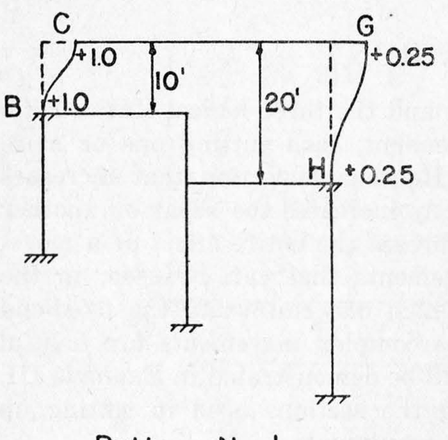


FIGURE 11a

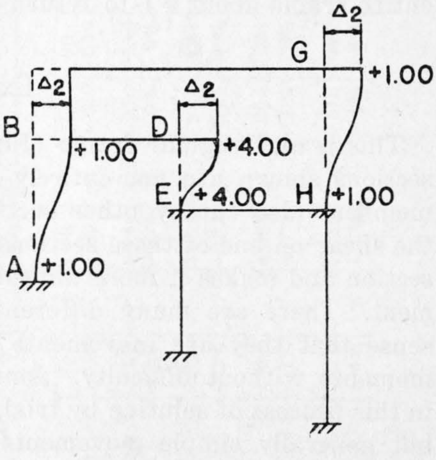


FIGURE 11b

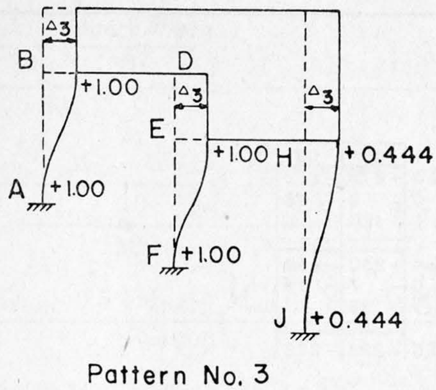


FIGURE 11c

$$M_{f,\,CB} = M_{f,\,BC} = \frac{+\,50 \times 10}{2} = +\,250 \text{ kf}$$

$$M_{f,\,GH} = M_{f,\,HG} = 0.25M_{f,\,BC} = +\,62.5 \text{ kf}$$

On section 3–3, the deflections shown in Fig. 11c lead to larger moments on BA and EF. A starting set of fixed-end moments might be based on assigning the entire 100 kip shear to these two members

$$M_{f,\,BA} = M_{f,\,AB} = M_{f,\,EF} = M_{f,\,EF} = \frac{100 \times 20}{4} = +\,500 \text{ kf}$$

$$M_{f,\,HJ} = M_{f,\,JH} = +\,222 \text{ kf}$$

On section 2–2, the deflections shown in Fig. 11b build up moment largely in DE. Some moment has already been put into BA and GH by the other two movements. Additional movement is necessary to build up a resistance to the 100 kip shear. This is taken, almost arbitrarily, as

$$M_{f,\,DE} = M_{f,ED} = +\,200 \text{ kf}$$

$$M_{f,\,BA} = M_{f,\,AB} = M_{f,\,GH} = M_{f,\,HG} = +\,50 \text{ kf}$$

For ordinary slide rule calculations without recording decimals, it is convenient to retain these relative moments but to double each value. These doubled values are recorded in Fig. 12, each identified by the pattern numbers shown in Fig. 11. Three cycles of moment distribution reduce the carry-over moments to reasonable size. Totals (excluding unbalanced carry-over moments) are then run for the first trial of the statics ratios.

Statics ratios will be set up as

$$SR = \frac{\text{Resisting shears on model section}}{\text{Corresponding external shear on frame}}$$

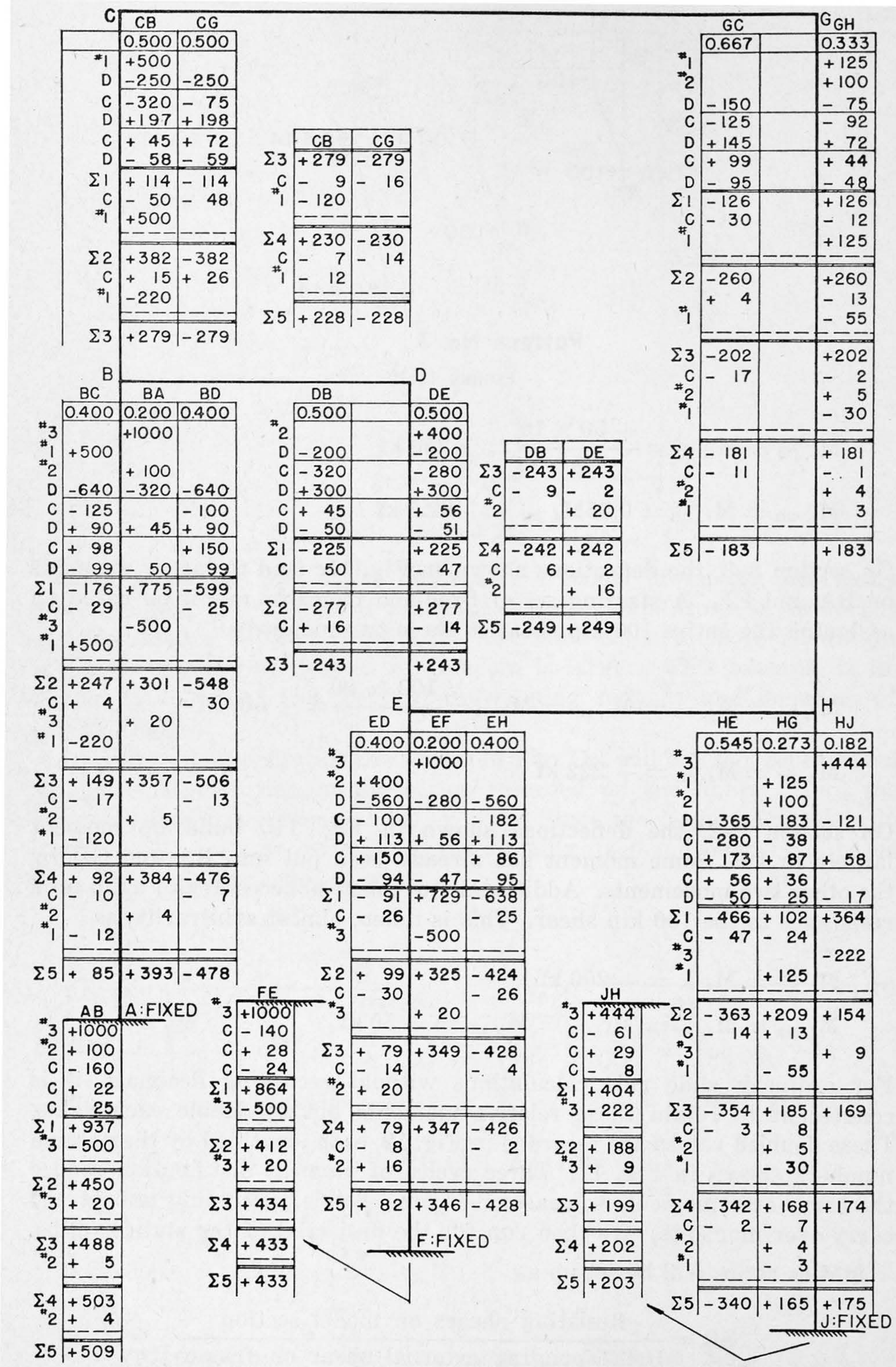

C
CB CG
0.500 0.500
#1 +500
D -250 -250
C -320 -75
D +197 +198
C +45 +72
D -58 -59
Σ1 +114 -114
C -50 -48
#1 +500
Σ2 +382 -382
C +15 +26
#1 -220
Σ3 +279 -279
CB CG
Σ3 +279 -279
C -9 -16
#1 -120
Σ4 +230 -230
C -7 -14
#1 -12
Σ5 +228 -228
G
GC GH
0.667 0.333
#1 +125
#2 +100
D -150 -75
C -125 -92
D +145 +72
C +99 +44
D -95 -48
Σ1 -126 +126
C -30 -12
#1 +125
Σ2 -260 +260
+4 -13
#1 -55
Σ3 -202 +202
C -17 -2
#2 +5
#1 -30
Σ4 -181 +181
C -11 -1
#2 +4
#1 -3
Σ5 -183 +183
B
BC BA BD
0.400 0.200 0.400
#3 +1000
#1 +500
#2 +100
D -640 -320 -640
C -125 -100
D +90 +45 +90
C +98 +150
D -99 -50 -99
Σ1 -176 +775 -599
C -29 -25
#3 -500
#1 +500
Σ2 +247 +301 -548
C +4 -30
#3 +20
#1 -220
Σ3 +149 +357 -506
C -17 -13
#2 +5
#1 -120
Σ4 +92 +384 -476
C -10 -7
#2 +4
#1 -12
Σ5 +85 +393 -478
D
DB DE
0.500 0.500
#2 +400
D -200 -200
C -320 -280
D +300 +300
C +45 +56
D -50 -51
Σ1 -225 +225
C -50 -47
Σ2 -277 +277
C +16 -14
Σ3 -243 +243
DB DE
Σ3 -243 +243
C -9 -2
#2 +20
Σ4 -242 +242
C -6 -2
#2 +16
Σ5 -249 +249
E
ED EF EH
0.400 0.200 0.400
#3 +1000
#2 +400
D -560 -280 -560
C -100 -182
D +113 +56 +113
C +150 +86
D -94 -47 -95
Σ1 -91 +729 -638
C -26 -25
#3 -500
Σ2 +99 +325 -424
C -30 -26
#3 +20
Σ3 +79 +349 -428
C -14 -4
#2 +20
Σ4 +79 +347 -426
C -8 -2
#2 +16
Σ5 +82 +346 -428
F:FIXED
H
HE HG HJ
0.545 0.273 0.182
#3 +444
#1 +125
#2 +100
D -365 -183 -121
C -280 -38
D +173 +87 +58
C +56 +36
D -50 -25 -17
Σ1 -466 +102 +364
C -47 -24
#3 -222
#1 +125
Σ2 -363 +209 +154
C -14 +13
#3 +9
#1 -55
Σ3 -354 +185 +169
C -3 -8
#2 +5
#1 -30
Σ4 -342 +168 +174
C -2 -7
#2 +4
#1 -3
Σ5 -340 +165 +175
J:FIXED
AB A:FIXED
#3 +1000
#2 +100
C -160
C +22
C -25
Σ1 +937
#3 -500
Σ2 +450
#3 +20
Σ3 +488
#2 +5
Σ4 +503
#2 +4
Σ5 +509
FE
#3 +1000
C -140
C +28
C -24
Σ1 +864
#3 -500
Σ2 +412
#3 +20
Σ3 +434
Σ4 +433
Σ5 +433
JH
#3 +444
C -61
C +29
C -8
Σ1 +404
#3 -222
Σ2 +188
#3 +9
Σ3 +199
Σ4 +202
Σ5 +203

FIGURE 12

$$SR_1 = \frac{\dfrac{M_{CB} + M_{BC}}{10} + \dfrac{M_{GH} + M_{HG}}{20}}{50} = \frac{M_{CB} + M_{BC} + 0.5\,(M_{GH} + M_{HG})}{500}$$

$$SR_2 = \frac{\dfrac{M_{BA} + M_{AB}}{20} + \dfrac{M_{DE} + M_{ED}}{10} + \dfrac{M_{GH} + M_{HG}}{20}}{100}$$

$$= \frac{M_{DE} + M_{ED} + 0.5\,(M_{BA} + M_{AB} + M_{GH} + M_{HG})}{1000}$$

$$SR_3 = \frac{\dfrac{M_{BA} + M_{AB}}{20} + \dfrac{M_{EF} + M_{FE}}{20} + \dfrac{M_{HJ} + M_{JH}}{30}}{100}$$

$$= \frac{M_{BA} + M_{AB} + M_{EF} + M_{FE} + 0.667\,(M_{HJ} + M_{JH})}{2000}$$

The first moment distribution totals from Fig. 12 then give the following values:

Trial 1:

$$SR_1 = \frac{+114 - 176 + 0.5\,(126 + 102)}{500} = + \frac{52}{500} = +0.104$$

$$SR_2 = \frac{+225 - 91 + 0.5\,(775 + 937 + 126 + 102)}{1000} = \frac{1104}{1000} = +1.104$$

$$SR_3 = \frac{+775 + 937 + 729 + 864 + 0.667\,(364 + 404)}{2000}$$

$$= \frac{3817}{2000} = 1.908$$

This is a very large spread. SR_3 will be lowered by adding -500 units of pattern #3 in Fig. 11c (half of the original amount of this pattern used, since SR_3 needs to be halved). When this set of fixed-end moments is written in the table, it is noted that SR_2 will also be lowered some, but this is temporarily ignored. SR_1 must be greatly increased; hence $+500$ units of pattern #1, Fig. 11a, is added (compared to an initial trial of equal amount). This was not made larger since it was noted from the moment distribution already made that the effect of the original pattern #1 moment was almost cancelled out by large distribution moments at B and H originating from pattern #3 moments written in there. Added pattern #3 moments are now of opposite sign and presumably will have an opposite effect. Two more cycles of moment distribution (details omitted on Fig. 12) now lead to:

Trial 2:

$$SR_1 = \frac{860}{500} = 1.720$$

$$SR_2 = \frac{986}{1000} = 0.986$$

$$SR_3 = \frac{1716}{2000} = 0.858$$

Corrections can now be tried somewhat in proportion to the effect of the last step. SR_1 was increased 1.62 units by + 500 units of pattern #1. To lower it now to 0.98, or by 0.74 units, requires approximately

$\frac{0.74}{1.62} \times 500 = 228$ units. Add − 220 units of pattern #1. In like fashion

a proportion for SR_3 would lead to approximately + 58 units of pattern #3. However, when the values of pattern #1 are written in it is noted that the large negative unbalanced moment at B and in some measure at H will cause a positive distribution moment on BA and HJ. Hence only + 20 units of pattern #3 is added.

The distribution through two more cycles leads to new statics ratios:

Trial 3:

$$SR_1 = \frac{622}{500} = 1.244$$

$$SR_2 = \frac{938}{1000} = 0.938$$

$$SR_3 = \frac{1873}{2000} = 0.936$$

Again proportions based on the last trial would indicate a need for about − 141 units of pattern #1. Some of this is already supplied by unbalanced carry-over moments of − 26 units on CB and − 10 on GH. Hence add only − 120 units of pattern #1. It is desirable not to disturb SR_2 and SR_3, but pattern #1 has already added − 60kf on GH. Cancel this general effect by adding + 5 units of pattern #2, which adds a total of + 60 on the three members of SR_2. (Since the members are of various lengths, this is a rather rough guess.) These values lead to new statics ratios:

Trial 4:

$$SR_1 = \frac{496}{500} = 0.993$$

$$SR_2 = \frac{939}{1000} = 0.939$$

$$SR_3 = \frac{1918}{2000} = 0.959$$

These are fairly close together and unbalanced carry-over moments might well be closely considered in further estimates. Try to bring ratios together at about 0.960. Again by proportion, add −12 units of pattern #1, temporarily ignoring the −25 units of carry-over moment. To balance −18 units of carry-over moment and also raise SR_2 numerator by 21 units, add +4 units of pattern #2, a total of +48kf of moment. This adds +8 units to GH of SR_1 which partially cancels the −25 units of carry-over moment on SR_1. When the distribution is complete, the statics ratios become:

Trial 5:

$$SR_1 = \frac{487}{500} = 0.974$$

$$SR_2 = \frac{956}{1000} = 0.956$$

$$SR_3 = \frac{1933}{2000} = 0.966$$

This is reasonably close. In a system this complex, it is a very good practice to check the results by starting anew with the summation of fixed-end moments. This was done and the difference in individual moments in only one case was as much as 3 units, which is about the usual agreement to be expected. This solution was further corrected by adding +2 units of pattern #2 and −4 units of pattern #1, which gave statics ratios of 0.970, 0.966, and 0.969. This work has not been shown except to list the results as the first line of Table II under the designation "final" for comparative purposes.

When statics ratios are nearly identical, it is not very important to recognize their small differences. However, differences can be recognized by using SR_1 on CB, SR_2 on DE, SR_3 on EF and HJ, these all being members cut only by a single section. Likewise the average of SR_1 and SR_2

TABLE II

RELATIVE ACCURACY USING DIFFERENT STATICS RATIOS

C — G

	CB	CG	SR	% SPREAD IN SR	MAX. ERROR IN MOMENTS ABSOLUTE (KF)	MAX. ERROR IN MOMENTS PERCENT		GC	GH
FINAL	+234	-234	0.970	0.4	0	0	FINAL	-191	+191
#5	+234	-234	0.974	1.9	3	0.8*∓	#5	-190	+190
	0%	0%						-0.5%	-0.5%
#4	+232	-232	0.993	5.6	7	2.1*∓	#4	-187	+187
	-0.9%	-0.9%						-2.1%	-2.1%
#3	+224	-224	1.244	29.7	27	7 *	#3	-185	+185
	- 4%	- 4%						- 3%	- 3%
#2	+222	-222	1.720	72	80	20 *	#2	-192	+192
	- 5%	- 5%						+0.5%	+0.5%

* Except on the small Moment M_{BC}
∓ Except on the small Moment M_{ED}

B

	BC	BA	BD
F	+86	+407	-493
#5	+87	+409	-496
	+1.2%	+0.4%	+0.6%
#4	+93	+404	-497
	+ 8%	-0.7%	+0.8%
#3	+120	+380	-500
	+40%	-7.0%	+1.4%
#2	+144	+327	-471
	+68%	-20%	-4.5%

D

	DB	DE	SR
FINAL	-260	+260	0.966
#5	-260	+260	0.956
	0%	0%	
#4	-257	+257	0.939
	-1.2%	-1.2%	
#3	-259	+259	0.938
	-0.4%	-0.4%	
#2	-281	+281	0.986
	+ 8%	+ 8%	

E

	ED	EF	EH	SR
FINAL	+ 89	+355	-444	0.969
#5	+ 86	+358	-444	0.966
	- 3%	+0.8%	0%	
#4	+ 84	+362	-446	0.959
	- 6%	+2.0%	+0.5%	
#3	+ 84	+372	-456	0.936
	- 6%	+ 5%	+2.8%	
#2	+101	+378	-479	0.858
	+13%	+ 6%	+ 8%	

H

	HE	HG	HJ
FINAL	-353	+171	+182
#5	-352	+171	+181
	-0.3%	0%	-0.6%
#4	-355	+174	+181
	+0.6%	+1.8%	-0.6%
#3	-349	+169	+180
	- 1.1%	-1.2%	- 1.1%
#2	-333	+154	+179
	- 6%	-10%	-1.7%

A

	AB
F	+529
#5	+530
	+0.2%
#4	+530
	+0.2%
#3	+520
	- 1.7%
#2	+488
	- 8%

F

	FE
FINAL	+446
#5	+448
	+0.4%
#4	+452
	+1.3%
#3	+463
	+ 4%
#2	+480
	+ 8%

J

	JH
FINAL	+210
#5	+210
	0%
#4	+210
	0%
#3	+212
	+1.0%
#2	+219
	+4.3%

on GH, and the average of SR_2 and SR_3 on AB would be logical. The moment on horizontal members would then be found by making joint moments balance.

This procedure has been followed in preparing Table II which compares the results obtained with various sets of statics ratios. It will be noted for every value except M_{BC} that the error is materially less on a percentage basis than is the spread in statics ratios used. (This percentage spread in statics ratios is based on the extreme spread divided by the average of the ratios of that trial.) M_{BC} is a relatively small moment in the vicinity of large moments; its absolute error is not serious; its percentage error is not significantly higher than the spread in statics ratios. In the writers' opinion a design based on trial 3 would be quite satisfactory, one based on trial 4 quite above any criticism.

Types of Movement to Be Used

There is absolutely no theoretical limitation upon the type of movement that is introduced at any stage of this process. To be useful the movement must be one for which correct moments can be written. The writing of these moments is the only way by which the correct continuity of the structure can be maintained. In other words, when new moments are added they must correspond to some possible deflection pattern. One does not need to evaluate this deflection numerically, but it is the key to the relative moments used. In the preceding example only three of many possible patterns were used. Undoubtedly some other pattern could be developed so as to shorten the trial process, but it is questionable whether it is worth while for a single analysis to invest much time in exploring such possibilities when a problem can be made to converge to satisfactory statics ratios by the use of relatively simple fixed-end moments such as those of Fig. 11.

In a tower where each panel is a trapezoid, moments corresponding to simple movements are somewhat complex to write and rather involved to use. Since this shape is a fairly common one, a special deflection pattern that has proven helpful will be developed. Such panels can be solved without this pattern and one of the chief reasons for including it here is to indicate the wide degree of freedom which is open in using the statics ratio method.

Deflection Patterns for Trapezoidal Panels

If the entire upper portion of a tower with sloping legs is deflected a distance $\triangle$ with respect to the lower part, without any rotation of joints, the horizontal members in the upper portion are all deformed as shown in Fig. 13. Since legs CD and C′D′ are assumed unchanged in length, points C and C′ move normal to the axis of these legs. This causes some vertical movement of C downward and C′ upward and makes

$$\begin{aligned}\triangle' &= \triangle \tan \phi_1 + \triangle \tan \phi_2 \\ &= \triangle\ (\tan \phi_1 + \tan \phi_2)\end{aligned}$$

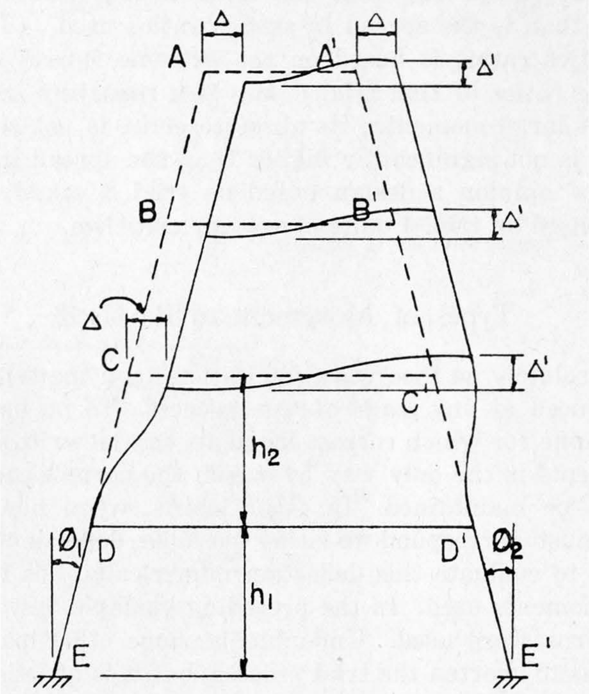


FIGURE 13

$\triangle'$ determines the fixed-end moment on CC′, and also on BB′ and AA′.

$$M_{f,\ CD} = M_{f,\ DC} = 6EK_{CD}\frac{\triangle \sec \phi_1}{L_{CD}} = 6EK_{CD}\frac{\triangle}{h_2}$$

$$M_{f,\ C'D'} = M_{f,\ D'C'} = 6EK_{C'D'}\frac{\triangle \sec \phi_2}{L_{C'D'}} = 6EK_{C'D'}\frac{\triangle}{h_2} = M_{f,\ CD}\frac{K_{C'D'}}{K_{CD}}$$

$$\begin{aligned}M_{f,\ CC'} = M_{f,\ C'C} &= -\ 6EK_{CC'}\frac{\triangle'}{L_{CC'}} = -\ 6EK_{CC'}\frac{\triangle\,(\tan \phi_1 + \tan \phi_2)}{L_{CC'}} \\ &= -\ M_{f,\ CD}\frac{K_{CC'}}{K_{CD}}\ \frac{h_2}{L_{CC'}}\ (\tan \phi_1 + \tan \phi_2)\end{aligned}$$

Similarly

$$M_{f,\ BB'} = -\ M_{f,\ CD}\frac{K_{BB'}}{K_{CD}}\ \frac{h_2}{L_{BB'}}\ (\tan \phi_1 + \tan \phi_2)$$

$$M_{f,\,BB'} = M_{f,\,CC'} \frac{K_{BB'}}{K_{CC'}} \frac{L_{CC'}}{L_{BB'}}$$

$$M_{f,\,AA'} = M_{f,\,CC'} \frac{K_{AA'}}{K_{CC'}} \frac{L_{CC'}}{L_{AA'}}$$

This is a very troublesome pattern. $M_{f,\,BB'}$ and $M_{f,\,AA'}$ are apt to be relalively large. The movement $\triangle$ as shown is one that might be used to correct a statics ratio cutting across CD and C′D′. The induced moments $M_{f,\,BB'}$ and $M_{f,\,AA'}$ when later distributed will probably seriously disturb the statics ratios in the higher panels. In other words, this pattern results in moments over too much of the frame. The pattern could be made more satisfactory by holding A and B undeflected as in Fig. 14. This would

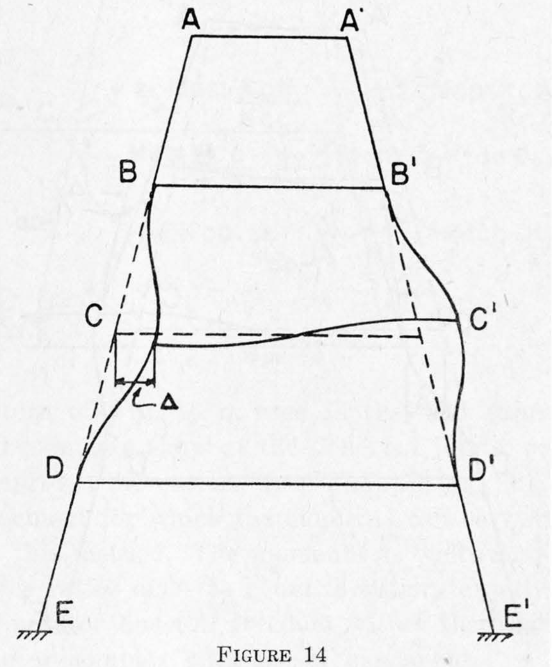


FIGURE 14

induce moments in legs BC and B′C′ of the same general order or magnitude as those in CD and C′D′. This involves a direct change in the statics ratio of panel BC as well as the one desired in panel DC. This is better than having panel AB also disturbed. This pattern is a practical one when BCD and B′C′D′ are originally straight, but for many cases the following more complex movement will give smaller moments in panel BC and hence cause less disturbance there. Also, BC and B′C′ can be different in slope from CD and C′D′ without any complication of the following pattern.

This movement will be visualized in two stages for convenience. In Fig. 13 imagine that BC is temporarily pin connected at C and B′C′ likewise at C′. When C is deflected $\triangle$ to the right, with A and B free to deflect without artificial restraint, the upper panels will rotate through a counter clockwise angle $\triangle'/L_{CC'}$ (Fig. 15), as determined by the movement of CC′. No distortion or moment will exist above CC′. The moments in CC′, CD, and C′D′ before joint rotation is permitted are the same as in Figs. 13 or 14. The continuity has been violated, however, by the fact that CB and C′B′ have each rotated through the counter-clockwise angle $\triangle'/L_{CC'}$ relative to joints C and C′. This continuity will now be restored by rotating end C of member CB through a clockwise angle of $\triangle'/L_{CC'}$ and rotating end C′ of C′B′ through the same angle, while A, B, A′ and B′ are held in the position of Fig. 15 without any other rotation. CB and C′B′ are thus fixed at B and B′ and build up moments as follows when rotated at C and C′.

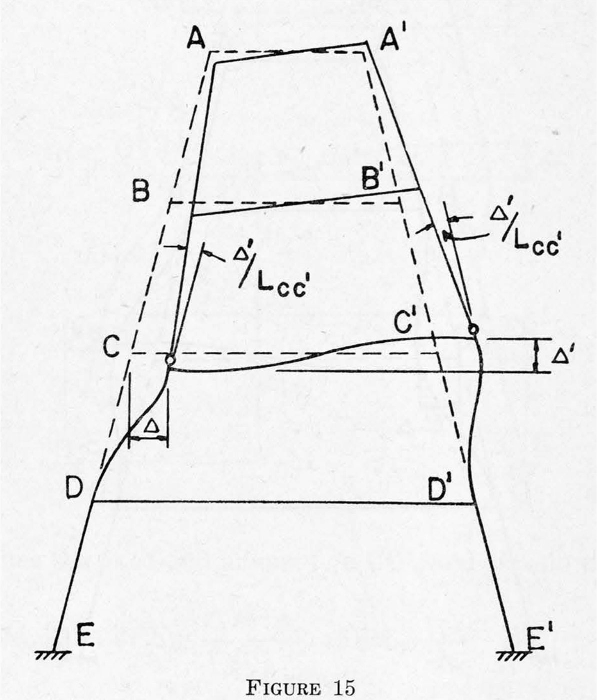


FIGURE 15

$$M_{f,\,CB} = 4EK_{CB}\theta_C = -4EK_{CB}\frac{\triangle'}{L_{CC'}} = +\frac{2}{3}M_{f,\,CC'}\frac{K_{CB}}{K_{CC'}}$$

$$M_{f,\,BC} = \frac{1}{2}M_{f,\,CB}$$

$$M_{f,\,C'B'} = -\,4EK_{C'B'}\frac{\triangle'}{L_{CC'}} = +\frac{2}{3}M_{f,\,CC'}\frac{K_{C'B'}}{K_{CC'}}$$

$$M_{f,\,B'C'} = \frac{1}{2}M_{f,\,C'B'}$$

This pattern of frame movement leads to the pattern of moments shown in Fig. 16 (B and B′ are fixed only in the sense that movements as outlined cause no moments in members beyond these points. Actually B and B′ have been both displaced and rotated as shown in Fig. 15.) In Fig. 16, C and C′ are displaced but not rotated. It should be noted that this pattern is independent of the initial slope of BC and B′C′; these members do not have to be straight line extensions of DC and D′C′. On the other hand, in Fig. 14 a change in leg slope in panel BC would complicate the pattern enormously, because B and B′ would then deflect vertically.

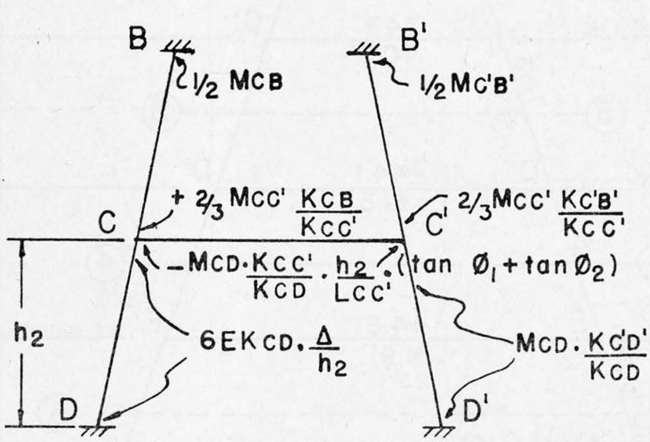


FIGURE 16

The basic pattern of Fig. 16 is used in the next example, but, when large unbalanced moments show at the deflected joints, rotation of these joints further improves the pattern (see Figs. 21 and 22). This rotation, or any other movement for which the moments can be written, is entirely permissible with this method. The moments as written must satisfy continuity; the statics ratios must be equal to satisfy exactly the conditions of statics. The operator has full freedom within these two conditions to use either simple or complex patterns of movement.

Example III

For the Kinzua Viaduct tower of Fig. 17, it is not easy to write shear equations directly, because horizontal shears involve components of the direct stresses in the legs. However, relatively simple equations can be written in terms of the summation of moments about point "O" where the legs produced would intersect. This is the device used to eliminate chord stresses in the analysis of ordinary sloping chord trusses. Statics ratios will be set up on this basis. Horizontal sections through each panel are logical here and will be numbered from the top for easy reference.

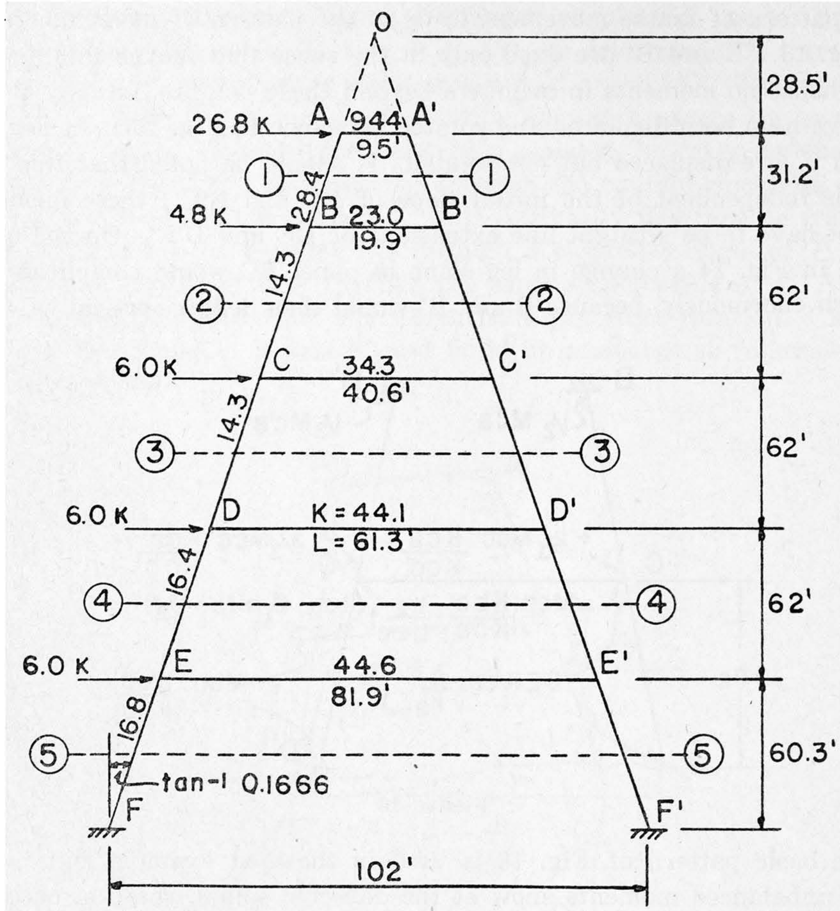


FIGURE 17

For the statics ratio on section 1–1, consider the section cut just above BB′ as in Fig. 18. All unknown quantities have been shown as though positive. This is the safest procedure. Due to symmetry of frame:

$$V_{BA} = V_{B'A'} = \frac{M_{BA} + M_{AB}}{31.6} \qquad\qquad M_{BA} = M_{B'A'}$$

Resisting moment about "O"

$$= 60.5\ (V_{BA} + V_{B'A'})\ - M_{BA} - M_{B'A'}$$

$$= 2\,\frac{60.5}{31.6}\ (M_{BA} + M_{AB})\ - 2M_{BA}$$

$$= 2\,(1.915M_{AB} + 0.915M_{BA})$$

Moment of external forces about "O"

$$= 26.8 \times 28.5 = 764 \text{ kf}$$

$$SR_1 = \frac{\text{Resisting moment}}{\text{External moment}} = \frac{2(1.915M_{AB} + 0.915M_{BA})}{764}$$

$$= \frac{M_{AB} + 0.478M_{BA}}{199.5}$$

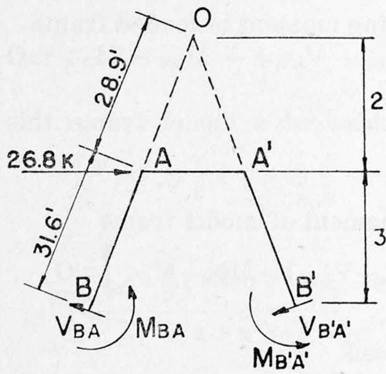


FIGURE 18

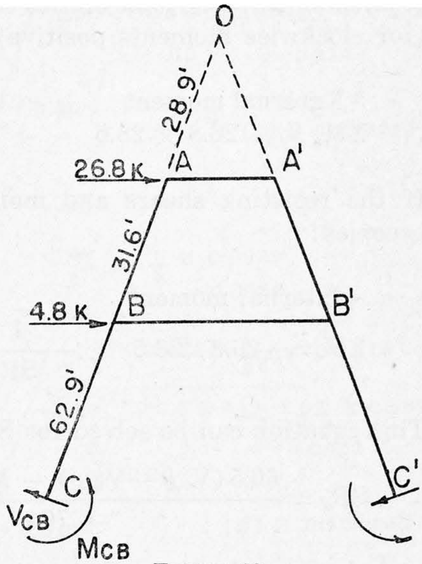


FIGURE 19

For the statics ratio on section 2–2, consider the section cut just above CC′ as in Fig. 19.

Resisting moment about "O"

$$= 2\left(\frac{M_{CB} + M_{BC}}{62.9}\right) 123.4 - 2M_{CB} = 2\,(1.961M_{BC} + 0.961M_{CB})$$

External moment $= 764 + 4.8 \times 59.7 = 1051$

$$SR_2 = \frac{2(1.961M_{BC} + 0.961M_{CB})}{1051} = \frac{M_{BC} + 0.490M_{CB}}{268}$$

In like fashion on lower sections:

$$SR_3 = \frac{M_{CD} + 0.662M_{DC}}{301}$$

$$SR_4 = \frac{M_{DE} + 0.748M_{ED}}{363}$$

$$SR_5 = \frac{M_{EF} + 0.803M_{FE}}{429}$$

Usually the statics ratios can be written in this fashion without encountering any uncertainties as to signs. Nevertheless, some may prefer a more formal approach for all problems and others may like it for special cases. The use of a formal equilibrium equation is recommended in such cases. For the section shown in Fig. 18, the final solution of the problem must satisfy the statics equation given by $\Sigma M_o = 0$. This equation (for clockwise moments positive) is:

$$\underset{\text{External moment}}{\Sigma M_o = -26.8 \times 28.5} \quad + \quad \underset{\text{Resisting moment of loaded frame}}{60.5(V_{BA} + V_{B'A'}) - M_{BA} - M_{B'A'} = 0}$$

If the resisting shears and moments are those of a model frame this becomes:

$$\underset{\text{External moment}}{\Sigma M_o = -26.8 \times 28.5} \quad + \underset{\text{Resisting moment of model frame}}{\frac{1}{SR_1}\left[60.5(V_{BA} + V_{B'A'}) - M_{BA} - M_{B'A'}\right] = 0}$$

This equation can be solved for SR_1 as follows:

$$SR_1 = \frac{60.5(V_{BA} + V_{B'A'}) - M_{BA} - M_{B'A'}}{+764}$$

which reduces to the same equation already found:

$$SR_1 = \frac{M_{AB} + 0.478 M_{BA}}{+199.5}$$

The signs automatically follow when a formal equilibrium equation is written in this fashion.

The patterns of Fig. 16 are too complex to carry in mind. Hence they are worked out separately for a movement of each joint, in this case (for convenience) sufficient to give a moment of 0.5 at each end of the lower leg. These patterns are recorded in Fig. 20. Due to symmetry of the tower, only one-half is recorded.

It is noted that the pattern for movement of joint A is not very satisfactory since the excessive moment in AA′ will distribute so as to alter M_{AB} greatly. This situation can be further improved by allowing joints A and A′ to rotate until in balance, a type of movement not yet used in this paper. Due to symmetry of the frame, all horizontal members take on a reverse curvature. They can be taken as terminal members by increasing their stiffness by 50%; and this leaves only half of the frame to analyze. In Fig. 21a, the pattern of Fig. 20a is modified by balancing joint A; and in Fig. 21b these results are proportionately adjusted for convenience to give $M_{AB} + M_{BA} = 1.00$. It will be noted that the resulting pattern of Fig. 21 is really for joint A deflected while wholly unrestrained in any manner. This pattern will be used in place of Fig. 20a.

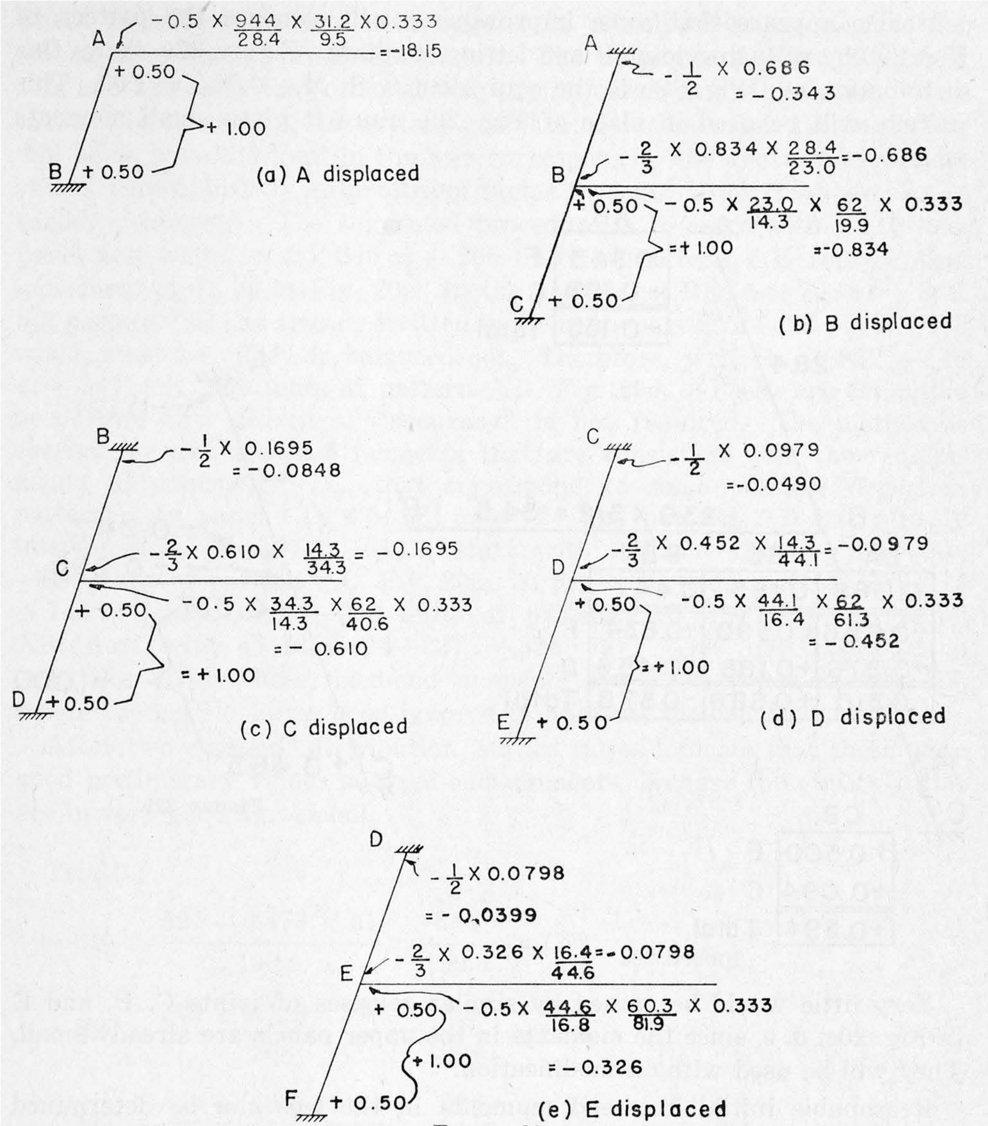


FIGURE 20

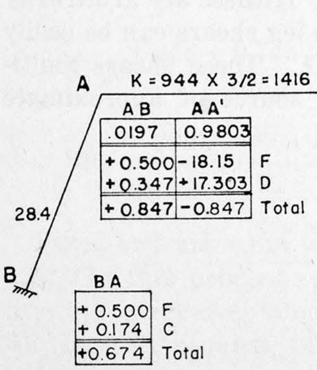


FIGURE 21a

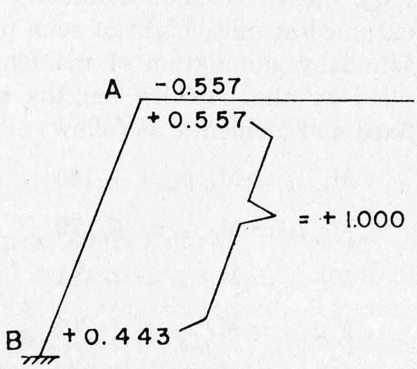


FIGURE 21b

It also appears that some improvement will come to the pattern of Fig. 20b by releasing joint B and letting it rotate. Figure 22a shows this distribution and Fig. 22b is the equivalent with $M_{BC} + M_{CB} = 1.00$. This pattern will be used in place of Fig. 20b since it gives small moments in AB.

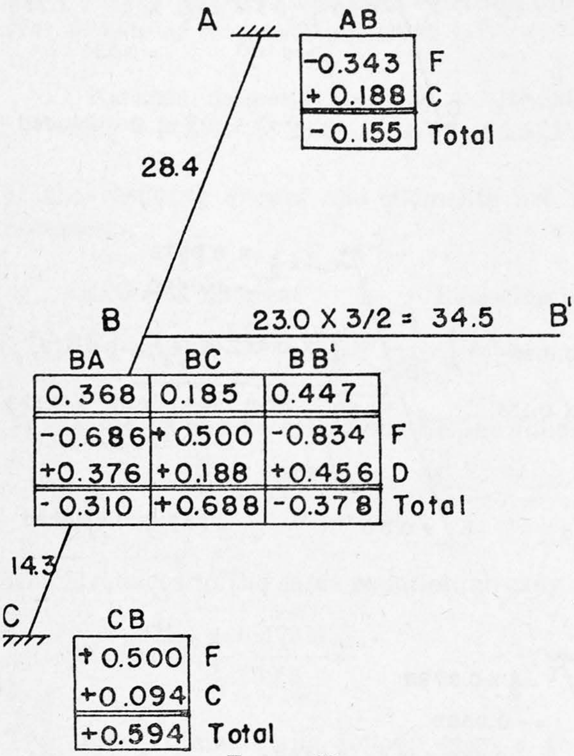


FIGURE 22a

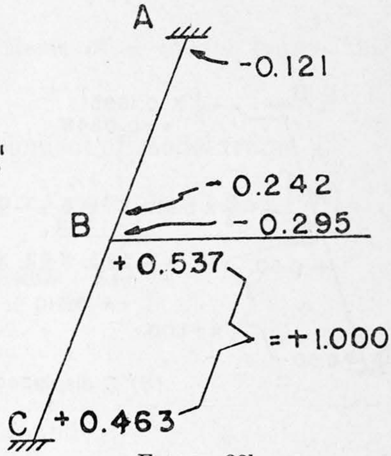


FIGURE 22b

Very little would be gained by similar releases of joints C, D, and E in Fig. 20c, d, e, since the moments in the upper panels are already small. They will be used without modification.

Reasonable initial fixed-end moments in the legs can be determined from shears based on assumed points of inflection. If these are arbitrarily assumed at mid-height of each panel, approximate leg shears can be easily found by summation of moments about point "O." These shears multiplied by the half leg lengths are the suggested source of approximate fixed-end moments, as follows:

$$M_{f,\,AB} = M_{f,\,BA} = +135$$

$$M_{f,\,BC} = M_{f,\,CB} = +179$$

$$M_{f,\,CD} = M_{f,\,DC} = +181$$

$$M_{f,\,DE} = M_{f,\,ED} = +208$$

$$M_{f,\,EF} = M_{f,\,FE} = +240$$

There is a complication with regard to establishing any desired leg moments in these trapezoidal panels. The patterns of Figs. 20c, d, e, 21b, and 22b show that other accompanying moments are necessary, some of them in other legs. In writing initial fixed-end moments, the objective will be to have the total in the legs correspond to the above approximate values (multiplied by an arbitrary factor 2 to get larger numbers, as in earlier problems). The suggested procedure is to start with the lowest panel and write in $4 \times 240 = +960$ units of pattern $\triangle E$ representing movement of E, as in Fig. 20e. In the next panel DE, $4 \times 208 = +832$, but pattern $\triangle E$ has already written in moments on DE of $-38-77=-115$ which must be offset or balanced out. Therefore, write in $+832+115 = +947$, say, 960 units of pattern $\triangle D$, Fig. 20d. (These are estimated needs and this degree of "accuracy" is not required. The method is correct for *any* assumed moments that are consistent with the requirements of continuity, i.e., that correspond to some possible deflection pattern.) In panel CD, $4 \times 181 = +724$ and pattern $\triangle D$ has already introduced $-47-94=-141$; start with $+724+141=+865$, say, $+860$ units of pattern $\triangle C$, Fig. 20c. In panel BC, start with $4 \times 179 + 73 + 146 = +935$, say, $+940$ units of pattern $\triangle B$, Fig. 22b. In panel AB, start with $4 \times 135 + 114 + 227 = +881$, say, $+880$ units of pattern $\triangle A$, Fig. 21b. These fixed-end moments are all recorded in Fig. 23, where the leg slope has been ignored in aligning the calculations.

After two cycles of distribution, statics ratios indicate that these were good preliminary values of fixed-end moments, because the statics ratios are in very good agreement.

Trial 1:

$$SR_1 = \frac{321 + 0.478 \times 51}{199.5} = \frac{345}{199.5} = 1.73$$

$$SR_2 = \frac{361 + 0.490 \times 244}{268} = \frac{480}{268} = 1.79$$

$$SR_3 = \frac{341 + 0.662 \times 281}{301} = \frac{526}{301} = 1.75$$

$$SR_4 = \frac{348 + 0.748 \times 286}{363} = \frac{561}{363} = 1.54$$

$$SR_5 = \frac{389 + 0.803 \times 434}{429} = \frac{736}{429} = 1.72$$

Fixed-end moments will be added in an attempt to bring all ratios to 1.72. In this process added moments will be considered in the light of existing carry-over moments that have not yet been balanced. To raise SR_4 by 0.18, requires that the numerator be raised 66 units; the existing unbalanced carry-over moments on DE total $+7$; add $+59$, say, $+60$ units

A

	0.0197	0.9803
ΔB	- 114	
ΔA	+ 490	- 490
D	+ 2	+ 112
C	- 58	
D	+ 1	+ 57
Σ1	+ 321	- 321

	0.0197	0.9803
Σ1	+ 321	- 321
C	+ 2	
ΔB	+ 4	
ΔA	- 8	+ 8
D	0	- 6
Σ2	+ 319	- 319

	0.0197	0.9803
Σ2	+ 319	- 319
C	0	
ΔB	+ 1	
ΔA	- 2	+ 2
D	0	- 1
Σ3	+ 318	- 318
Final	+ 185	- 185
Amirikian	+184.8	-184.8

B

	0.368	0.185	0.447
ΔC		- 73	
ΔB	- 227	+ 504	- 277
ΔA	+ 390		
D	- 117	- 59	- 141
C	+ 1	- 13	
D	+ 4	+ 2	+ 6
Σ1	+ 51	+ 361	- 412

	0.368	0.185	0.447
Σ1	+ 51	+ 361	- 412
C	0	+ 5	
ΔB	+ 7	- 16	+ 9
ΔA	- 7		
D	+ 1	0	+ 1
Σ2	+ 52	+ 350	- 402

	0.368	0.185	0.447
Σ2	+ 52	+ 350	- 402
C	0	+ 1	
ΔC		+ 1	
ΔB	+ 2	- 4	+ 2
ΔA	- 1		
D	0	0	- 1
Σ3	+ 52	+ 349	- 401
Final	+ 30	+ 202	- 232
Amirikian	+ 31.3	+202.5	-232.8

C

	0.178	0.178	0.644
ΔD		- 47	
ΔC	- 146	+ 430	- 525
ΔB	+ 436		
D	- 26	- 26	- 96
C	- 30	- 26	
D	+ 10	+ 10	+ 36
Σ1	+ 244	+ 341	- 585

	0.178	0.178	0.644
Σ1	+ 244	+ 341	- 585
C	+ 1	+ 4	
ΔD		- 3	
ΔB	- 14		
D	+ 2	+ 2	+ 8
Σ2	+ 233	+ 344	- 577

	0.178	0.178	0.644
Σ2	+ 233	+ 344	- 577
C	0	0	
ΔD		- 1	
ΔC	+ 2	- 5	+ 6
ΔB	- 4		
D	+ 1	0	+ 1
Σ3	+ 232	+ 338	- 570
Final	+ 134	+ 196	- 330
Amirikian	+134.7	+195.0	-329.7

D

	0.148	0.169	0.683
ΔE		- 38	
ΔD	- 94	+ 480	- 434
ΔC	+ 430		
D	- 51	- 58	- 235
C	- 13	- 46	
D	+ 9	+ 10	+ 40
Σ1	+ 281	+ 348	- 629

	0.148	0.169	0.683
Σ1	+ 281	+ 348	- 629
C	+ 5	+ 2	
ΔD	- 6	+ 30	- 27
D	0	- 1	- 3
Σ2	+ 280	+ 379	- 659

	0.148	0.169	0.683
Σ2	+ 280	+ 379	- 659
C	+ 1	- 3	
ΔE		0	
ΔD	- 2	+ 10	- 9
ΔC	- 5		
D	+ 1	+ 1	+ 6
C	0	- 1	
D	0	0	+ 1
Σ3	+ 275	+ 386	- 661
Final	+ 159	+ 223	- 382
Amirikian	+158.1	+223.2	-381.3

E

	0.164	0.168	0.668
ΔE	- 77	+ 480	- 313
ΔD	+ 480		
D	- 93	- 96	- 381
C	- 29		
D	+ 5	+ 5	+ 19
Σ1	+ 268	+ 389	- 675

	0.164	0.168	0.668
Σ1	+ 268	+ 389	- 675
C	+ 5		
ΔD	+ 30		
D	- 6	- 6	- 23
Σ2	+ 315	+ 383	- 698

	0.164	0.168	0.668
Σ2	+ 315	+ 383	- 698
C	0		
ΔE	- 1	+ 5	- 3
ΔD	+ 10		
C	0		
D	- 2	- 2	- 7
Σ3	+ 322	+ 386	- 708
Final	+ 186	+ 225	- 411
Amirikian	+187.6	+225.8	-413.4

F Fixed

ΔE	+ 480
C	- 48
C	+ 2
Σ1	+ 434

Σ1	+ 434
C	+ 2
C	- 3
Σ2	+ 433

Σ2	+ 433
ΔE	+ 5
C	- 1
Σ3	+ 437
Final	+ 254
Amirikian	+254.8

FIGURE 23

of pattern $\triangle D$. (This might well have been a little more since the numerator uses only $0.748M_{ED}$ and there is also some shrinkage from distribution.) This is entered in the calculation form. SR_3 is only a trifle high and will not be modified yet, since carry-over moments and pattern $\triangle D$ moments just added on CD total only -9 and will not change the ratio seriously. SR_2 needs a reduction of 19 units in addition to the cancelling of $+6$ units already on hand, a total of 25 units; say, add -30 units of pattern $\triangle B$. SR_1 is about right but add -15 units of pattern $\triangle A$ to reduce the $+13$ units already written for AB.

The new statics ratios that result after one cycle of distribution are:

Trial 2:

$$SR_1 = \frac{344}{199.5} = 1.72$$

$$SR_2 = \frac{465}{268} = 1.73$$

$$SR_3 = \frac{529}{301} = 1.755$$

$$SR_4 = \frac{615}{363} = 1.69$$

$$SR_5 = \frac{730}{429} = 1.70$$

The objective of 1.72 still appears as good as any. For SR_5 to increase 0.02, the numerator must increase $+8$ units; add $+10$ units of pattern $\triangle E$. For SR_4 to increase 0.03, the numerator must increase $+11$ units; -4 units already exist; add $+20$ units of pattern $\triangle D$. For SR_3 the numerator should decrease 11 units; -2 units exist; add -10 units of pattern $\triangle C$. For SR_2 the numerator should decrease 3 units; $+4$ units exist; add -8 units of pattern $\triangle B$. For SR_1 no change is wanted but $+3$ units exist; add -3 units of pattern $\triangle A$.

After balancing completely, the statics ratios become:

Trial 3:

	As run	Recap. values
$SR_1 =$	$\frac{343}{199.5} = 1.719$	$\frac{343}{199.5} = 1.719$
$SR_2 =$	$\frac{463}{268} = 1.728$	$\frac{462}{268} = 1.724$
$SR_3 =$	$\frac{520}{301} = 1.728$	$\frac{520}{301} = 1.728$

TABLE III

Relative Accuracy Using Different Statics Ratios

A

	AB	AA	S.R.	Spread in S.R. Overall	Spread in S.R. Adjacent	Max. Error in Moments Absolute	Max. Error in Moments Percent
Amirikian	+184.8	-184.8					
Recap.	+185	-185	1.719	1.0%	1.0%	Assumed Zero	
#3	+185	-185	1.719	1.0%	1.0%	1	0.5%
Diff.	0%	0%					
#2	+185	-185	1.72	3.8%	3.8%	2	0.7% (Except 3.3% on M_{ba})
Diff.	0%	0%					
#1	+186	-186	1.73	15%	12%	5	1.5% (Except 3.3% on M_{ba})
Diff.	+0.5%	+0.5%					

B

	BA	BC	BB'	S.R.
Amirikian	+31.3	+202.5	-233.8	
Recap.	+31	+202	-233	1.724
#3	+31	+202	-233	1.728
Diff.	0%	0%	0%	
#2	+30	+202	-232	1.73
Diff.	-3.3%	0%	-0.4%	
#1	+30	+202	-232	1.79
Diff.	-3.3%	0%	-0.4%	

C

	CB	CD	CC'	S.R.
Amirikian	+134.7	+195.0	-329.7	
Recap.	+134	+196	-330	1.728
#3	+134	+197	-331	1.728
Diff.	0%	+0.5%	+0.3%	
#2	+135	+196	-331	1.755
Diff.	+0.7%	0%	+0.3%	
#1	+136	+195	-331	1.75
Diff.	+1.5%	-0.5%	+0.3%	

D

DC	DE	DD'

D

	DC	DE	DD'	S.R.
Amirikian	+158.1	+224.3	-382.4	
Recap.	+159	+223	-382	1.730
#3	+159	+224	-383	1.727
Diff.	0%	+0.4%	+0.3%	
#2	+159	+224	-383	1.69
Diff.	0%	+0.4%	+0.3%	
#1	+161	+226	-387	1.54
Diff.	+1.3%	+1.3%	+1.3%	

E

	ED	EF	EE'	S.R.
Amirikian	+187.6	+225.8	-413.4	
Recap.	+187	+225	-412	1.713
#3	+186	+225	-411	1.718
Diff.	-0.5%	0%	-0.2%	
#2	+186	+225	-411	1.70
Diff.	-0.5%	0%	-0.2%	
#1	+186	+226	-412	1.72
Diff.	-0.5%	+0.4%	0%	

F

	FE
Amirikian	+254.8
Recap.	+254
#3	+254
Diff.	0%
#2	+255
Diff.	+0.4%
#1	+252
Diff.	-0.8%

$$SR_4 = \frac{627}{363} = 1.727 \qquad \frac{628}{363} = 1.730$$

$$SR_5 = \frac{738}{429} = 1.718 \qquad \frac{735}{429} = 1.713$$

The second set of the above values, headed "Recap. values," is based on a separate distribution starting with the *total* units of each pattern used. It is a better set of values because its calculations involved fewer small fragments. However, the differences between the two sets of statics ratios are negligible in this case and only the original solution has been included here. The final moments are shown on the calculation sheet, the various statics ratios having been used for the legs at their respective height and the horizontal member moment being that required to balance the joints. For comparison, values found by Amirikian by a modified slope deflection process[3] are also tabulated. (Signs of these values have been made to conform to the notation of this paper and beam moments have been added as required by joint equilibrium.) The agreement is quite satisfactory, the greatest difference being 2 units for M_{CD}. It is noted that part of this particular difference is probably due to the fact that Amirikian's moments in this panel are about 0.5% too low to balance the static forces. Other solutions for this bent (for half these loads) are also available.[4, 5]

Table III compares the accuracy obtained from these several sets of statics ratios, the per cent difference that is shown being based on the final solution. Amirikian's results are also shown in case the reader prefers to use them as a reference. It should be noted that even the first statics ratios lead to good results in this example. It is inherent in this method that even early statics ratios lead to resisting moments of just the right *total* amount to balance the external forces. Such errors as exist are due to the improper distribution of these resisting moments to the two ends of members cut by a section or improper distribution between two members cut by the same section. Some error in distribution also exists from use of different statics ratios on adjacent sections which is actually a violation of continuity at joints between the sections.

Loads Between Panel Points

Loads between panel points can be included directly in this type of analysis only if statics ratios are made to take on the special value of

[3]Amirikian, "Analysis of Rigid Frames" (1942), p. 121.

[4]"A Rapid and Concise Method of Analyzing Rigid Viaduct Bents," L. C. Maugh, Eng. News-Record, Vol. 114 (Mar. 14, 1935), p. 379.

[5]"The Kinzua Viaduct of the Erie Railroad Company," C. R. Grimm, Member ASCE, Trans. ASCE, Vol. XLVI (1901), pp. 21–77.

unity.* It is much better in most cases to set up a preliminary simple moment distribution in which no lateral or vertical movement of joints is permitted. This will require that unknown restraining forces be applied at each point where joint deflection could occur. After the distribution process is complete, these restraints can be evaluated.* A separate analysis for deflection or sway of joints can then be run by the general method here proposed, using as external forces these restraining forces *reversed in direction.* If the original restraining forces act to the left, the frame tends to move to the right and this is the reason for this reversal in direction. Those who are accustomed to analyzing one-story frames by moment distribution will recognize this as the usual procedure for simple side-sway problems. The total moments in the frame are the sum of the moments for the two analyses, the one for loads between joints without sway or joint deflection, the other for the effect of sway or deflection.

As a comment on the flexibility of this general method, it might be noted that the first analysis above may be made to include any arbitrary side-sway or deflection allowance that may be estimated to represent the effects of the loads, or of settlement, or of expansion, etc. In such a case the second analysis furnishes the *additional* side-sway or deflection that is needed for complete equilibrium.

Conclusions

The method of statics ratios is a convenient method for analyzing frames involving deflections. It is particularly advantageous as the number of degrees of freedom of movement of the frame increases. It has never failed to yield a satisfactory solution for any type of frame where the authors have tried it. The Vierendeel truss with unequal chords and chords of varying slope becomes a routine problem under this procedure. It is believed to be a general method for any practical frame and loading.

The apparent accuracy secured is limited solely by the number of significant places carried in the tabulations and the closeness with which statics ratios are made to agree. The real accuracy is usually much more limited by assumptions of loading, by neglect of the modified deformations present in the joints, the neglect of change in axial length of members, the neglect of shear deformations, and other factors, all of these

*A special statics ratio value of unity makes the procedure very nearly the same as that commonly known by the name successive approximations.

The calculation of restraining forces is really an unnecessary step in most frame problems. A Circular, "Equilibrium Equations without Restraining Forces for Moment—Distribution—Sway Problems," by Phil M. Ferguson, soon to be published by the Bureau of Engineering Research, indicates how problem solutions are often made somewhat simpler by avoiding this calculation. The same procedure can easily be adapted to statics ratio problems involving loads between panel points.

being limitations that are common to most current methods of frame analysis. In view of these conditions engineers may well shorten the solution of many problems by stopping when only a reasonable agreement of statics ratios is attained. The accuracy thus obtained may be estimated from the comparisons tabulated in this paper. In significant values of moment the percentage errors will usually be considerably smaller than the per cent spread in adjacent statics ratios.

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‡By error printed as Bulletin No. 2831.